ASSIGNMENT SET – I

Mathematics: Semester-III

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-305A

Paper: Special Paper-OR: Advanced Optimization

1.	***a)*** Find the conjugate directions of the following real symmetric matrix: $\begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix}$ ***b)*** Is it possible to obtain the optimal integer solution of an IPP after neglecting integer restrictions and round-off the optimal solution of the corresponding LPP? Justify. ***c)*** “Revised simplex method is better than the original simplex method”, why? ***d)*** What are the basic differences between analytical and numerical optimization methods? ***e)*** Define goal programming problem. ***f)*** In Branch and bound method, when a node is called “***fathomed***”? ***g)*** Define the term “***Gomory’s constraint***”. ***h)*** Define integer programming problem? Give an example of it. ***i)*** Write the limitation of Fibonacci Method? ***j)*** Define quadratically convergent method and A-conjugate directions.	2 Marks for each question

	k) Explain Different types of achievements in goal programming problem. ***l)*** Define unimodal maximization and minimization function. ***m)*** Using algebraic approach show that the expression $ax + \frac{b}{x} + c; a, b > 0$ $as\ minimum\ value\ 2\sqrt{ab} + c\ at\ x = \sqrt{\frac{b}{a}}$. ***n)*** What is post optimality analysis? ***o)*** State the necessary and sufficient conditions for maximum point of a multivariable optimization problem. ***p)*** Differentiate revised simplex and dual simplex approaches. ***q)*** Explain deletion of an existing variable in the optimal table of an LPP. ***r)*** What is Unimodal Function? ***s)*** What is basic difference between Fibonacci method and Golden section method? Which one is better and why? ***t)*** What is the basic difference between direct search method and decent method? ***u)*** Write the iteration scheme of steepest descent method.	
2.	***a)*** Describe the Golden section method to optimize a unimodal function and implement a flowchart of this method. ***b)*** Minimize the function $f(x) = 0.65 - \left[\frac{0.75}{1+x^2}\right] - 0.65\,x\tan^{-1}\left(\frac{1}{x}\right)$in the interval [0, 3] by Fibonacci method using $n = 6$. ***c)*** Derive the conditions of the range of discrete changes of the component of cost vector (C) of the LPP $Maximize\ Z = CX$ subject to A$X = b$ and $X \geq 0$ such that the optimal solution does not alter. ***d)*** The optimal result of the LPP $Maximizez = 2x_1 + 2x_2$ $subjectto$ $5x_1 + 3x_2 \leq 8$	4 Marks for each question

$$x_1 + 2x_2 \le 4$$
$$and x_1, x_2 \ge 0$$

is given in the following table:

C_B	X_B	B	y_1	y_2	y_3	y_4
2	x_1	4/7	1	0	2/7	-3/7
2	x_2	12/7	0	1	-1/7	5/7
$Z_j - C_j$			0	0	2/7	4/7

Find the optimal results after addition of the following constraints:

I. $3x_1 + 2x_2 \le 6.$

II. $3x_1 + 3x_2 \le 5.$

e) Write the procedure of Fibonacci method to solve a unimodal optimization problem.

f) Find the 1st Gomory's constraints of the following integer programming problem

Maximize z= $3x_1 - 2x_2$

Subject to $12x_1 + 7x_2 \le 28$ $x_1, x_2 \ge 0$ and are integers.

g) The production manager facts the problem of job allocation among three of his teams. The processing rates of three teams are 5, 6, and 8 units per hour respectively.The normal Working hours for each team are 8 hours per day. The Production manager has the following goals for the next day in order of priority:

(i) The manager wants to avoid any underachievement of production level, which is set at 180 units of production.

(ii) Any overtime operation of team 2 beyond 2hrs and team 3 beyond 3 hrs. should be avoided.

(iii) Minimize the sum of overtime.

Formulate above goal programming problem.

h) Using Newton's method

Minimize $f(x_1, x_2) = x_1 - x_2 + 2{x_1}^2 + 2x_1x_2 + {x_2}^2$ with (0,0) as starting point.

i) When required an artificial constraint method to solve an LPP. Explain

	it with an example. ***j)*** write the steps of Davidon – Fletcher –Powell method to solve a non-linear optimization problem.	
3.	***a)*** Solve the following LPP using Revised Simplex method. Max z = x_1+2x_2 Subject to, $2x_1+5x_2\geq 6$, $x_1+x_2 \geq 2$, $x_1, x_2 \geq 0$. ***b)*** Using Davidon-Fletcher-Powell method minimize $f(x_1,x_2) = x_1^2 + 2x_2^2 + x_1 - 2x_2$ starting from the point $\begin{pmatrix}1\\0\end{pmatrix}$. ***c)*** Solve the following IPP using Branch and bound method. $Maximize \quad z = 5x_1 + 4x_2$ $Subject\ to \quad x_1 + x_2 \leq 5,\ 10x_1 + 6x_2 \leq 45,\ x_1, x_2 \geq 0,\ x_1, x_2\ integers.$ ***d)*** Solve the following goal programming problem: $Minimizez = P_1 d_1^- + P_2(2d_2^- + 3d_3^-)$ $subject\ to$ $20x_1 + 10x_2 \leq 60$ $10x_1 + 10x_2 \leq 40$ $40x_1 + 80x_2 + d_1^- - d_1^+ = 600$ $x_1 + d_2^- - d_2^+ = 2$ $x_2 + d_3^- - d_3^+ = 2$ $x_1, x_2, d_i^-, d_i^+ \geq 0, i = 1, 2, 3$ ***e)*** Solve the following problem using Gomory's cutting plane method: $Maximize \quad f = 4x_1 + 3x_2$ $Subject\ to \quad 3x_1 + 4x_2 \leq 12,\ 4x_1 + 2x_2 \leq 9,\ x_1, x_2 \geq 0\ and\ integers.$ ***f)*** Solve the following IPP using Branch and bound method. Max $z = 7x_1 + 9x_2$	8 Marks for each question

Subject to $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$ $x_1, x_2 \geq 0$ and integers.

g) Solve the following LPP by revised simplex method

Minimize $z = 2x_1 + x_2$

Subject to constraints

$3x_1 + x_2 \leq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$ and $x_1, x_2 \geq 0$

h) Using cutting plane method, solve

Maximize f = $7 - 2x_1 - 4x_2$

Subject to the constraints

$(x_1 - 4)^2 + 2(x_2 - 3)^2 - 12 \leq 0$

$x_1 + 2x_2 - 6 \leq 0$

$1 \leq x_1, x_2 \leq 6$ with the tolerance $\varepsilon = 0.03$

i) Using steepest descent method minimize the function $f(x_1,\ x_2,\ x_3) = {x_1}^2 + {x_2}^2 + {x_3}^2 - 6x_1 - 4\,x_2 + 3\,x_3 + 9$ starting from the point$(1,\ 2, -3)$.

j) Determine the effect of discrete changes in the requirement vector of the LPP $Max\ z = cx,\ subject\ to\ Ax = b, x \geq 0$.

k) Define goal programming problem. A firm produces two products A and B. Each product must be processed through two departments namely 1 and 2. Department 1 has 30 hours of production capacity per day and department 2 has 60 hours. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2. Management has established the following goals it would like to achieve in determining the daily product mix:

P_1: The joint total production at least 10 units.

P_2: Producing at least 7 units of product B.

P_3: Producing at least 8 units of product A.

Formulate this problem as a goal programming problem.

	l) Using this method $minimize\ f = x_1{}^2 + x_2{}^2 + x_3{}^2 + 2gx_1 + 2h\,x_2 + 2k\,x_3 + c$ starting from the point $(1,\ 0,\ 1)$. ***m)*** Solve the following IPP by Gomory's cutting plane method $$Minimize\ \ z = 2x_1 + \ 3x_2$$ subject to the constrains $80\,x_1 + 31\,x_2 \geq 248$ $x_1,\ x_2\ \geq 0$ and are integers.	

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